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Original Article

Bayesian Vector Autoregression (BVAR) modeling and forecasting of the dynamic interrelationship between GDP and agricultural sector in Nigeria

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Abstract
In the Bayesian VAR literatures, the Litterman Prior has been compared with other priors for example with Sims-Zha prior. It has been shown that the Sims-Zha prior has more advantages over the Litterman prior. For example, the Litterman prior estimates the VAR coefficients on equation-by-equation basis, but the Sims-Zha prior estimates the parameter for the full system in a multivariate regression. The implication is that the Sims-Zha prior allows for a more general specification and produce a tractable multivariate normal posterior distribution. We proposed four (4) versions of BVAR models of the secondary data on GDP and Agriculture sector for the Nigerian economy collected from CBN website from 1960 to 2011. We found that the BVAR1 produced the forecast with the minimum RMSE and MAE as 0.05666357 and 0.03721166 respectively. We therefore concluded that from the economic point of view, our results suggested that in the presence of prior information (which can come from different sources, either experience or economy theory) can significantly improved the forecasts from economic models.

1. Introduction

One of the major advantages of the reduced form multiple equation time series modeling such as VARs is their applications to forecasting and policy analysis [1]. In recent times, it has been discovered that unrestricted VAR models tend to overfit the data, attribute unrealistic portions of the variance in time series to their deterministic components, and overestimate the magnitude of the coefficients of distant lags of variables as a result of sampling error [2-4]. Because of the many problems encountered in using unrestricted VARs, that gave rise to Bayesian Econometrics and Bayesian Time Series Analysis.

The Bayesian VAR (BVAR) were originally devised to improve macroeconomic forecast [5-7]. In addition, the Bayesian method was intended to solve the problems associated with unrestricted VAR models. The advantages of BVAR includes: they make in-sample fitting less dramatic and improve out-of-sample performances. These many advantages of BVAR made it more useful in forecasting short-term macro-economic series both in Central Banks and other international financial institutions.

Bayesian approach has been especially effective in dealing with specification uncertainty inherent in time-series modeling. The final strength of the BVAR has been the emergence of a consistent method for specifying the Bayesian prior, including formal statistical criteria for examining the performance of alternative specifications [8]. Another advantage of BVAR is that it does not ponder too much on any of the parameters of the model, but rather, emphasis is laid on the use of prior distribution for the parameters. The prior distributions are the key factor in the BVAR approach. Another feature of the Bayesian VAR framework, it allows for the presence of trend in the variables [4].

The aim of this present work is to proposed BVAR models for modeling and forecasting GDP and Agriculture sector in Nigeria. Our work is motivated by the work of Adenomon and Oyejola, [9](2013) who found that Agricultural sector contribute more to the GDP than any other sector of the Nigerian economy. This work is also motivated by the recent rebasing of the GDP by the CBN and NBS that the Agricultural sector contributes more to the National development of the Nigeria economy.

2. Model Specification

2.1 Bayesian Vector Autoregression with Sims-Zha prior

The most popular BVAR model is that of the Litterman (1986) [10]. However, in recent times, the BVAR model of Sims and Zha [6](1998) has gained popularity both in economic time series and political analysis. As stated in Brandt and Freeman [2] (2006), the Litterman proposed BVAR for the reduced form of the model, while Sims-Zha specified prior for the simultaneous equation of the model. They further noted that Sims-Zha has more advantage compared to the BVAR proposed by Litterman. The Sims-Zha BVAR allows for a more general specification and can produce a tractable multivariate normal posterior distribution. Again, for the Litterman BVAR, the estimation of the VAR coefficients is done on an equation-by-equation basis as in the reduced form version while the Sims-Zha BVAR estimates the parameters for the full system in a multivariate regression. Also In terms of forecasting, the Sims-Zha BVAR performs as well as or better than models of commercial forecasters.

The procedure for BVAR with Sims-Zha prior is as follows. We considered the following (identified) dynamic simultaneous equation model as

$$\sum_{l=0}^{p} \underset{1\times m}{y_{t-l}} \underset{m\times m}{A_l} = \underset{1\times m}{d} + \underset{1\times m}{\varepsilon_t} \qquad ; t = 1,2,\ldots T$$

This is an m-dimensional VAR for a sample of size T with y_t a vector of observations at time t, A_l the coefficient matrix for the l^{th} lag; p the maximum number of lags (assumed known), d a vector of constant and ε_t a vector of i.i.d normal structural shocks such that

$$E[\varepsilon_t / y_{t-s}, s>0] = \underset{1\times m}{0} \text{ and } E[\varepsilon_t' / y_{t-s}, s>0] = \underset{m\times m}{I}$$

The structural model can be transformed into a multivariate regression by defining A_0 as the contemporaneous conditions of the series and A_+ as a matrix of the coefficients on the lagged variables by $YA_0 + XA_+ = E$ where Y is Txm, A_0 is mxm, X is Tx(mp+1), A_+ is (mp+1)xm and E is Txm.

To define the VAR in a compact form

$$a_0 = vec(A_0),\ a_+ = vec\begin{bmatrix} -A_1 \\ . \\ . \\ . \\ -A_p \\ d \end{bmatrix}, A = \begin{pmatrix} A_0 \\ A_+ \end{pmatrix},\ a = vec(A)$$

The VAR model can then be written as a linear projection of the residual by letting Z=[Y X], and $A = [A_0 / A_+]'$ is a conformable stacking of the parameters in A_0 and A_+:

$YA_0 + XA_+ = E$

$ZA = E.$

In order to derive the Bayesian estimator for this structural equation model, we have to examine the (conditional) likelihood function for normally distributed residuals

$$L(Y/A) \propto |A_0|^T \exp[-0.5tr(ZA)'(ZA)]$$
$$\propto |A_0|^T \exp[-0.5a'(I \otimes Z'Z)a].$$

The prior overall of the structural parameters has the form

$$\pi(a) = \pi(a_+ / a_0)\pi(a_0)$$
$$\pi(a) = \pi(a_0)\phi(\tilde{a}_+, \Psi)$$

$\tilde{a}_+$ denotes the mean parameters in the prior for a_+, Ψ is the prior covariance for $\tilde{a}_+$ and $\phi(\,)$ is a multivariate normal density.

The posterior for the coefficients is then

$$q(A) \propto L(Y/A)\pi(a_0)\phi(\tilde{a}_+, \Psi)$$
$$\propto \pi(a_0)|A_0|^T |\psi|^{-0.5} \times \exp[-0.5(a_0'(I \otimes Y'Y))a_0$$
$$-2a_+'(I \otimes X'Y)a_0 + a_+'(I \otimes X'X)a_+ + \tilde{a}_+'\Psi\tilde{a}_+)]$$

The posterior is conditional multivariate normal, since the prior has a conjugated form. In this case, the posterior can be estimated by a multivariate seeming unrelated regression (SUR) model. The forecast and inferences can be generated by exploiting the multivariate normality of the posterior distribution of the coefficients. The normal conditional prior for the mean of the structural parameters is given by $E(A_+ / A_0) = \begin{bmatrix} A_0 \\ 0 \end{bmatrix}$ while the prior covariance matrix for a given by $V(A_+ / A_0) = \Psi$ though complicated, it is specified to reflect the following general beliefs and facts about the series being model:

1) The standard deviations around the first lag coefficients are proportionate to all the other lags.
2) The weight of each variable's own lags is the same as those of other variables' lags.
3) The standard deviations of the coefficients of longer lags are proportionately smaller than those on the earlier lags. (Lag coefficients shrink to zero over time and have smaller variance at higher lags.)
4) The standard deviation of the intercept is proportionate to the standard deviation of the residuals for the equation.
5) The standard deviation of the sums of the autoregressive coefficients should be proportionate to the standard deviation of the residuals for the respective equation (consistent with the possibility of cointegration).
6) The variance of the initial conditions should be proportionate to the mean of the series. These are "dummy initial observations" that capture trends or beliefs about stationarity and are correlated across the equations. The summary of the Sims-Zha prior is given in table 1 below.

Table 1: Hyperparameters of Sims-Zha reference prior

Parameter	Range	Interpretation
λ_0	[0,1]	Overall scale of the error covariance matrix
λ_1	>0	Standard deviation around A_1 (persistence)
λ_2	=1	Weight of own lag versus other lags
λ_3	>0	Lag decay
λ_4	≥0	Scale of standard deviation of intercept
λ_5	≥0	Scale of standard deviation of exogenous variable coefficients
μ_5	≥0	Sum of coefficients/Cointegration (long-term trends)
μ_6	≥0	Initial observations/dummy observation (impacts of initial conditions)
v	>0	Prior degrees of freedom

Source: Brandt and Freeman, (2006)[2]

Then each diagonal element of Ψ therefore corresponds to the variance of the VAR parameters. The variance of each of these coefficients is assumed to have the form

$\psi_{l,j,i} = \left(\frac{\lambda_0\lambda_1}{\sigma_j l^{\lambda_3}}\right)^2$ for the element corresponding to the l^{th} lag of variable j in equation i.

The overall coefficient covariances are scaled by the value of error variances from m univariate AR(p) OLS regressions of each variable on its own lagged values, σ_j^2 for j=1, 2, . . .m. The parameter λ_0 sets an overall tightness across the elements of the prior on $\Sigma = A_0^{-1'} A_0^{-1}$. The hyperparameter λ_1 controls the tightness of the beliefs about the random walk prior or the standard deviation of the first lags. The l^{λ_3} term allows the variance of the coefficients on higher order lags to shrink as the lag length increases. The constant in the model receives a separate prior variance of $(\lambda_0\lambda_4)^2$ and the prior variance on any exogenous variables is $(\lambda_0\lambda_5)^2$. The Sims-Zha prior adds dummy observations to account for unit roots, trends, and cointegration which was not possible with the Litterman prior.

Given the reduced form model

$$y_t = c + y_{t-1}B_1 + . . . + y_{t-p}B_p + u_t$$

where $c = dA_0^{-1}$, $B_l = -A_l A_0^{-1}$, $l = 1,2,...p$, $u_t = \varepsilon_t A_0^{-1}$ and $\Sigma = A_0^{-1'} A_0^{-1}$

The matrix representation of the reduced form is given as

$$\underset{T\times m}{Y} = \underset{T\times(mp+1)}{X}\ \underset{(mp+1)\times m}{\beta} + \underset{T\times m}{U},\ U \sim MVN(0, \Sigma)$$

We can then construct a reduced form Bayesian SUR with the Sims-Zha prior as follows. The prior means for the reduced form coefficients are that B_1=I and B_2, . . . B_p=0. We assume that the prior has a conditional structure that is multivariate Normal-inverse Wishart distribution for the parameters in the model. To estimate the coefficients for the system of the reduced form model with the following estimators

$$\hat{\beta} = (\Psi^{-1} + X'X)^{-1}(\Psi^{-1}\bar{\beta} + X'Y)$$
$$\hat{\Sigma} = T^{-1}(Y'Y - \hat{\beta}'(X'X + \Psi^{-1})\hat{\beta} + \bar{\beta}'\Psi^{-1}\bar{\beta} + \bar{S})$$

where the Normal-inverse Wishart prior for the coefficients is

$\beta / \Sigma \sim N(\bar{\beta}, \Psi)$ and $\Sigma \sim IW(\bar{S}, v)$

This representation translates the prior proposed by Sims and Zha form from the structural model to the reduced form Brandt and Freeman; Sims and Zha and Sims. [2,6,7,11,12]

3. Materials and Methods

The time series data were collected for GDP and Agriculture Sector for the Nigerian Economy. The time series annual data span from 1960 to 2011 and the data were sourced from the Central Bank of Nigeria website. It is common in practice to divide the time series data into two parts: a training sample and a test sample [13]. In this work the training sample will be from 1960 to 2006 (T=32) while the test sample will be from 2007 to 2011. In the analysis, all series have been transformed into natural logarithms; this is in line with most econometrics analysis [14]. All the analyses are carried out with R software.

3.1 Setting of Hyperparameters for BVAR Model with Sims-Zha Prior

For the BVAR model with Sims-Zha prior, we will consider the following range of values for the hyperparameters given below and the Normal-Inverse Wishart prior.

We consider two tight priors and two loose priors as follows:

The Tight priors are as follows

$BVAR1 = (\lambda_0 = 0.6, \lambda_1 = 0.1, \lambda_3 = 1, \lambda_4 = 0.1, \lambda_5 = 0.07, \mu_5 = \mu_6 = 5)$

$BVAR2 = (\lambda_0 = 0.8, \lambda_1 = 0.1, \lambda_3 = 1, \lambda_4 = 0.1, \lambda_5 = 0.07, \mu_5 = \mu_6 = 5)$

The Loose priors are as follows

$BVAR3 = (\lambda_0 = 0.6, \lambda_1 = 0.15, \lambda_3 = 1, \lambda_4 = 0.15, \lambda_5 = 0.07, \mu_5 = \mu_6 = 2)$

$BVAR4 = (\lambda_0 = 0.8, \lambda_1 = 0.15, \lambda_3 = 1, \lambda_4 = 0.15, \lambda_5 = 0.07, \mu_5 = \mu_6 = 2)$

Where nμ is prior degrees of freedom given as m+1 where m is the number of variables in the multiple time series data. In our work nμ is 3 (that is two (2) time series variables plus 1(one)).

Our choice of Normal-Inverse Wishart prior for the BVAR models follow the work of Kadiyala & Karlsson [15], (1997) that Normal Wishart prior tends to performed better when compared to other priors. In addition Sims and Zha [6] in 1998 proposed Normal-Inverse Wishart prior because of its suitability for large systems while Breheny [16] in 2013 reported that the most advantage of wishart distribution is that it guaranteed to produce positive definite draws. Our choice of the overall tightness $\lambda_0 = 0.6$ and 0.8 is in line with work of Brandt, Colaresi and Freeman; Adenomon and Oyejola.[17-19]

4. Analysis and Discussion of Results

In this section we proposed four (4) versions of BVAR model with Sims-Zha prior as stated above denoted as BVAR1, BVAR2, BVAR3 and BVAR4. The BVAR models were implemented using the current version of this package (MSBVAR 0.7-2) in R which includes functionality to build and evaluate VAR, BVAR and BSVAR models with Markov Switching [20]. In this research work, we will use familiar measures such as Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) to gauge the difference in BVAR models [21]. The BVAR model with the minimum RMSE and MAE will emerge as the preferred model. The information criteria (AIC, BIC and HQ) choose lag 1 for the BVAR models (see Table 2A)

Table 2A: var. lag. specification (Training sample, lagmax=10)

	Lags	Log-Det	Chi^2	p-value
[1,]	10	-8.791196	6.052423	0.1952673
[2,]	9	-8.412920	2.436824	0.6559829
[3,]	8	-8.277541	3.474936	0.4816996
[4,]	7	-8.103794	2.504456	0.6438381
[5,]	6	-7.989955	3.938795	0.4143524
[6,]	5	-7.825839	4.238634	0.3746725
[7,]	4	-7.662814	2.737232	0.6027154
[8,]	3	-7.565056	2.237594	0.6921528
[9,]	2	-7.490470	1.316804	0.8585180
[10,]	1	-7.449319	0.000000	0.0000000
$results				
	Lags	**AIC**	**BIC**	**HQ**
[1,]	1	-7.124995*	-6.863765*	-7.032899*
[2,]	2	-6.949929	-6.514546	-6.796436
[3,]	3	-6.808299	-6.198763	-6.593409
[4,]	4	-6.689841	-5.906152	-6.413554
[5,]	5	-6.636650	-5.678806	-6.298965
[6,]	6	-6.584550	-5.452553	-6.185468
[7,]	7	-6.482172	-5.176023	-6.021694
[8,]	8	-6.439703	-4.959400	-5.917827
[9,]	9	-6.358866	-4.704410	-5.775593
[10,]	10	-6.520926	-4.692317	-5.876256

s*Minimum value

Table 2B: The Actual Series and the Forecast of the Model of the dynamic interrelationship between GDP and Agricultural sector in Nigeria from 2007 to 2011

			Forecasts							
Year	Actual series		BVAR1		BVAR2		BVAR3		BVAR4	
	lngdp	lnagric	lngdp	lnagric	lngdp	lnagric	lngdp	lnagric	lngdp	lnagric
2007	16.8436	15.7262	16.885346	15.736063	16.907143	15.756843	16.998469	15.843306	17.004574	15.849513
2008	17.0058	15.8926	17.033233	15.876021	17.077219	15.917959	17.262278	16.093176	17.274684	16.105800
2009	17.0228	16.0340	17.182343	16.017143	17.248916	16.080622	17.530163	16.346928	17.549069	16.366182
2010	17.3414	16.1487	17.332687	16.159437	17.422249	16.244846	17.802186	16.604622	17.827797	16.630726
2011	17.4410	16.2657	17.484274	16.302915	17.597235	16.410647	18.078412	16.866320	18.110937	16.899497

Table 2C: **The RMSE and MAE of the forecast of the forecasting Models on the dynamic interrelationship between GDP and Agricultural sector in Nigeria**

	Models			
Forecasts statistics	**BVAR1**	**BVAR2**	**BVAR3**	**BVAR4**
RMSE	0.05666357	0.11198394	0.4107325	0.4320174
MAE	0.03721166	0.09409542	0.3703135	0.3896053

In Tables 2B and 2C presented the forecasts and the forecast statistics for the BVAR models respectively. We found that the BVAR1 stands out as the preferred BVAR model for the Nigerian economic with the following hyperparameters:

$(p = 1, \lambda_0 = 0.6, \lambda_1 = 0.1, \lambda_3 = 1, \lambda_4 = 0.1, \lambda_5 = 0.07, \mu_5 = \mu_6 = 5)$.

The preferred BVAR1 model produces the minimum RMSE and MAE as 0.05666357 and 0.03721166 respectively.

5. Conclusion and recommendation

This study from the economic point of view concluded that the results suggested that the presence of prior information (which can come from different sources, either experience or economy theory) can significantly improved the forecasts from economic model especially the BVAR model. This work therefore recommends that the BVAR1 model can be used to obtain better forecast in the future for GDP and Agriculture variables in Nigeria.

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